WS 2011-2012 12.12.2011

Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 9

Exercise 1: [Termination]

Prove the following theorem:

Let A be a set, $>$ a total well-founded ordering on A and I a function mapping every k-ary function symbol f to a mapping $I(f) : A^k \to A$, strictly monotonously increasing in every argument (i.e. for all $a_1, \ldots, a_k \in A, i \in \{1, \ldots, k\}$, and $a_i > a$ holds: $I(f)(a_1, \ldots, a_i, \ldots, a_k) > I(f)(a_1, \ldots, a_{i-1}, a, a_{i+1}, \ldots, a_k)$.
Let $I(\beta) : \mathrm{Term}(F, V) \to A$ be defined as:

$$I(\beta)(t) = \beta(t), \text{ if } t \in V$$
$$I(\beta)(f(t_1, \ldots, t_n)) = I(f)(I(\beta)(t_1), \ldots, I(\beta)(t_n)).$$

Let G be a term-rewriting system and let $I(\beta)(l) > I(\beta)(r)$ for every rule $l \to r \in G$ and for every variable assigment $\beta : V \to A$. Then G is terminating.

Exercise 2: [Example for termination]

Consider the rule system $R : f(x) \to h(s(x)), h(0) \to h(s(0))$ with $x \in V$. Prove:

1. The theorem of exercise 1 is not applicable to R for $A = \mathbb{N}$.
2. R is confluent.
3. R is terminating.

Exercise 3: [Completion]

Let $E = \{x + 0 = x, x + s(y) = s(x + y), x + p(y) = p(x + y), x - 0 = x, x - s(y) = p(x - y), x - p(y) = s(x - y), s(p(x)) = x, p(s(x)) = x, ((x + y) - x) = y, (x + (y - x)) = y, ((x - y) + y) = x\}$

1. Complete E using any reduction ordering you like.

   Be verbose, write down for at least 5 most general unificators how you determined them when looking for critical pairs. Write down all critical pairs, you have looked at.

   Hint: Start with CPs of the last three equations.
2. Show, that completion will not succeed. Make a suggestion, what can be done on this problem.

Exercise 4: [Completion modulo $\sim$]

Let $>$ be a Knuth-Bendix-ordering with weight function φ defined by $\varphi(s) = 1$ for $s \in F \cup V$.

Let $E = \{f(x+y) \to f(x) * f(y), f(0) \to 1, x+0 \to x, 0+x \to x, x*1 \to x, 1*x \to x\}$ and $G = \{x+y = y+x, (x+y)+z = x+(y+z), x*y = y*x, (x*y)*z = x*(y*z)\}$.

Complete E modulo G with respect to $>$.

Delivery: until 18.12.2011,
by E-Mail to huechting@informatik.uni-kl.de