Exercises to the Lecture FSVT

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Exercise 1: [Confluence and termination of rule sets over ground terms]

Let $R = \{(l_k, r_k) | k = 1, \ldots, n)\}$ be a finite rule set over ground terms. Prove:

1. If there is an infinite chain, then there is a rule $(l, r) \in R$ with an infinite chain from r.
2. If there is an infinite chain, then there is a j with $1 \leq j \leq n$ and a ground term t, such that $l_j \overset{+}{\Rightarrow} t$ and l_j is a subterm of t.
3. Termination of R is decidable. (Termination is often denounced as 'Kettenbedingung' in german literature.)
4. Develop sufficient conditions for local confluence.

Exercise 2: [Knuth-Bendix-ordering]

Let $\varphi : F \cup V \to \mathbb{N}$ be a weight function with

$$\begin{aligned} \varphi(x) &= \alpha > 0 && \text{for all } x \in V && (1)\\ \varphi(f) &\geq \alpha && \text{if } f \text{ 0-ary} && (2)\\ \varphi(f) &> 0 && \text{if } f \text{ 1-ary} && (3)\\ \varphi(f) &\geq 0 && \text{else} && (4) \end{aligned}$$

Extend φ to $\varphi : \mathrm{Term}(F, V) \to \mathbb{N}$ by

$$\varphi(f(t_1, \ldots, t_n)) = \varphi(f) + \sum_{i=1,\ldots,n} \varphi(t_i)$$

Define $s > t$ iff. $\varphi(s) > \varphi(t)$ and $|s|_x \geq |t|_x$ for all $x \in V$. Then $>$ is called a Knuth-Bendix-ordering. Prove for any Knuth-Bendix-ordering $>$:

1. $>$ is strict part of a wellfounded partial ordering
2. $>$ is compatible with substitution
3. $>$ is compatible with term replacement

Exercise 3:

Let

$$\begin{aligned} R_1 &= \{F(0, 1, x) \to F(x, x, x)\}\\ R_2 &= \{G(x, y) \to x, G(x, y) \to y\}. \end{aligned}$$

1. Prove: R_1 and R_2 are terminating.
2. Prove or disprove: The rule set $R_1 \cup R_2$ is terminating.

Exercise 4: [Example confluence and critical pairs]

Consider the rule system $R : h(x, f(x)) \to c, h(x, x) \to b, k(x) \to x, g(a) \to f(g(k(a)))$.

1. Prove: There are no critical pairs of R.
2. Prove: R is not confluent.
3. Why is there no contradiction?

Exercise 5: [Local coherence and critical pairs]

Prove: Let $\mathrm{CP}(R, G)$ be defined as the set of critical pairs regarding R and the set of equations G oriented in both ways. If R is left-linear, then the following statements are equivalent.

1. $\to_R$ is locally coherent modulo $\sim$.
2. For every critical pair $(t_1, t_2) \in \mathrm{CP}(R, G)$ holds $t_1 \downarrow_\sim t_2$.

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