Illustrating the ASM Function Classification

- A real time CLOCK:
  - Monitored: CurrTime: Real (supposed to be increasing)
  - Controlled: DisplayTime: Nat x Nat
  - Static: Delta: Real (system dependent time granularity), +, conversion to convert Real values into elements of Nat

If DisplayTime + Delta = CurrTime
Then DisplayTime := conversion(CurrTime)

- With the following derived fct
  - ClockTick = 1 iff (CurrTime = DisplayTime + Delta)

expressing a standard computing procedure, the rule becomes

If ClockTick = 1 Then DisplayTime := CurrTime

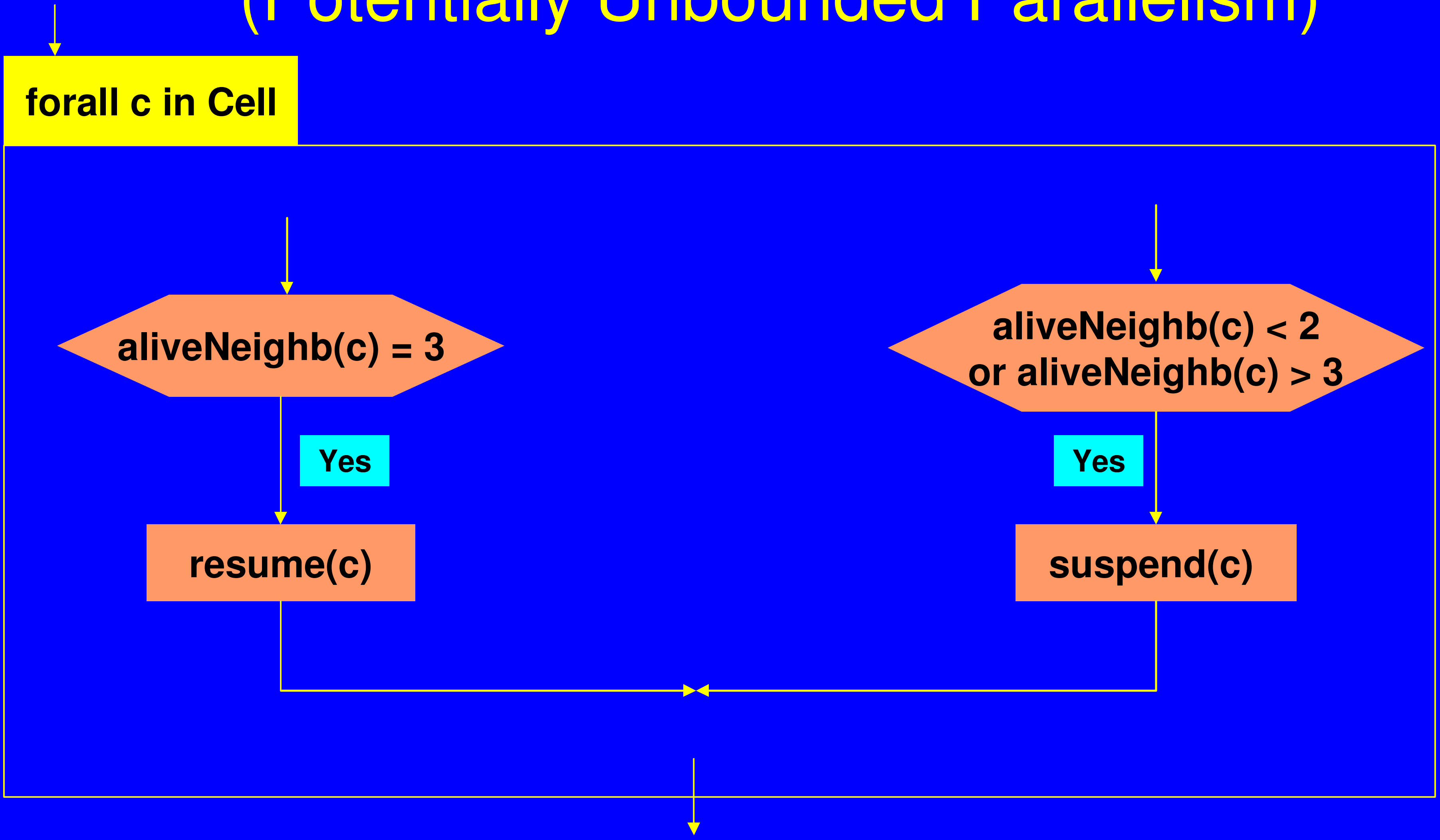
Bounded Synchronous Parallelism

- $\text{CycleThru}(R_0,..,R_n)$ =

```
forall i=0,…,n
    If    cycle = i
    Then
          R_i
          cycle := cycle+1 (mod n+1)
```

- Special case: Alternate(R,S) (n=1)

Conway's game of life: cell evolution rule (Potentially Unbounded Parallelism)



resume(c) = alive(c) := true

suspend(c) = alive(c) := false

Non-deterministic Sorting or Variable Assignment

- Non-deterministic sorting by iterating local swap:

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choose i,j in dom(a) s.t.i < j & a(i) > a(j)
    a(i) := a(j)
    a(j) := a(i)
```

- Non-deterministic choice of variable assignments in COLD:

```
ColdModify(Var) =
    choose n∈ N
        choose x(1),…,x(n) ∈ Var , choose v(1),…,v(n) ∈ Value
            forall i=1,…,n
                val(x(i)) := v(i)
```

Non-deterministic language generation (1)

- generating all and only the pairs $vw \in A^*$ of different words v,w of same length (i.e. $v \neq w$ and $|v| = |w|$)

```
choose n, i with $i < n$
        choose a,b $\in A$ with $a \neq b$
                v(i) := a
                w(i) := b
        forall $j < n$, $j \neq i$
                choose a,b $\in A$
                        v(j) : = a
                        w(j) := b
```

When all possible choices are realized, the set of reachable states vw of this ASM, started with (say) a b for some $a \neq b$, is the set of all vw with $v \neq w$ and $|v| = |w|$.

The power of non-determinism

- Let $L_n = \{ v \# w \mid v \neq w \text{ and } |v| = |w| = n\}$.
- Exercise. Show that for each n, L_n can be accepted by a non-deterministic finite automaton with $O(n^2)$ states.
- Every unambiguous automaton that accepts L_n needs at least 2^n states.
  - See C. M. R. Kintala and K-Y Pun and D. Wotschke: Concise representations of regular languages by degree and probabilistic finite automata. In: Math. Systems Theory 26 (4) 1993, 379—395.

Non-deterministic language generation (2)

- generating the words over alphabet $\{0,1\}$ of length at least n with a 1 in the n-th place, i.e. the words of form $v1w \in \{0,1\}^{n-1}\ 1\ \{0,1\}^*$.
- Let n be arbitrary, but fixed.

```
choose v ∈ {0,1}^(n-1)
choose w ∈ {0,1}*
        out := v1w
```

NB. When all possible choices are realized, the set of words appearing as values of out is the desired set.

For each n, there is a non-deterministic FSM with O(n) states which accepts the set $\{0,1\}^{n-1}\ 1\ \{0,1\}^*$, but every deterministic FSM accepting this set has at least 2^n states.

Double Linked Lists : Desired Operations

- Define an ASM which offers the following operations, predicates and functions on double linked lists, whose elements have values in a given set VALUE:
  - CreateList (VALUE) : create a new double linked list with elements taking values in Value
  - Append (L, Val) : append at the end a new element with given value
  - Insert (L, Val, Elem) : insert after Elem in L a new element with Val
  - Delete (L, Elem) : delete Elem from L
  - AccessByValue (L, Val) : return the first element in L with Val
  - AccessByIndex (L, i) : return the i-th element in L
  - empty (L), length (L), occurs (L, Elem), position (L, Elem)
  - Update (L, Elem, Val) : update the the value of Elem in L to Val
  - Cat (L1,L2) : concatenate two given lists in the given order
  - Split (L, Elem, L1, L2) : split L into L1, containing L up to including Elem, and L2 containing the rest list of L

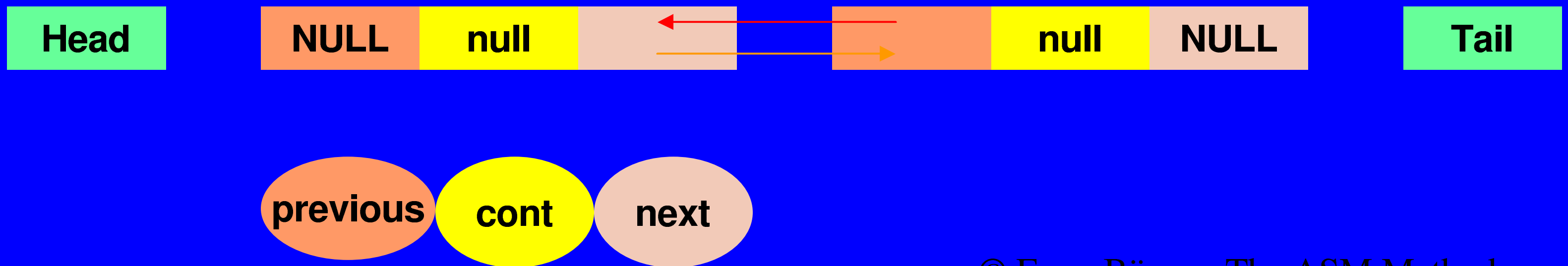
Double Linked Lists : Desired Properties

- Prove that the Linked List ASM has the following properties:
  - If the next-link of a list element Elem points to Elem’, then the previous-link of Elem’ points to Elem.
  - L is empty iff the next-link of its head points to its tail.
  - The set ELEM (L) of elements occurring in a list is the set of all E which can be reached, starting from the list head, by applying next-links until the list tail is encountered.
  - After applying Append (L, Val), the list is not empty.
  - A newly created linked list is empty and its length is 0.
  - By Append/Delete the list length in/de-creases by 1.
  - For non empty L and arbitrary elements E the following holds:

    Append (Delete (L,E),E) = Delete (Append (L,E),E)

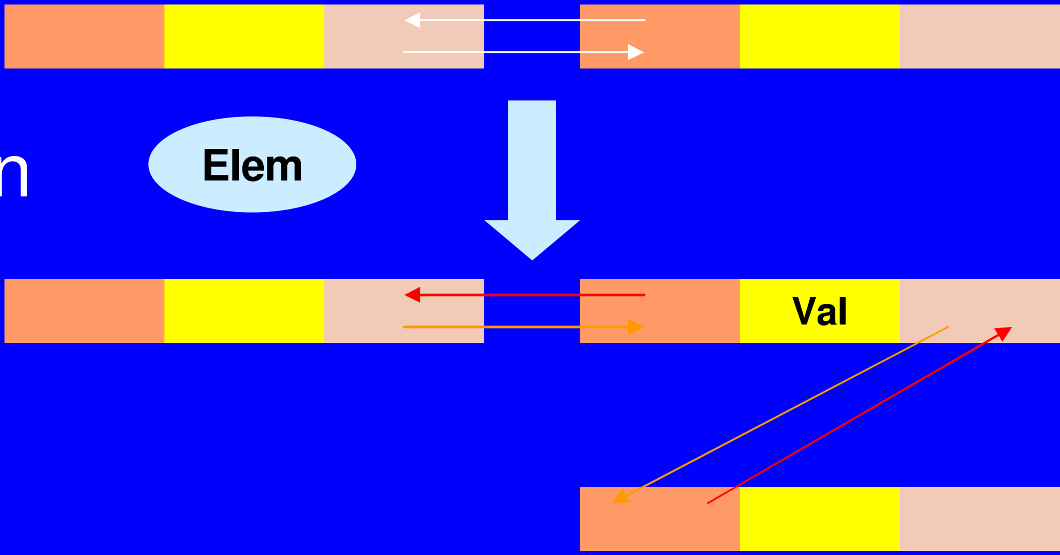
Double Linked Lists : Signature

- LINKED-LIST (VALUE) : dynamic set, with fcts “pointing” to structures of the following form (often VALUE suppressed) :
  - dynamic set ELEM (L) of “objects” currly listed in L
  - distinguished elems Head (L), Tail (L) ∈ ELEM (L)
  - previous (L), next (L): ELEM (L) → ELEM (L)   dyn link fcts
  - cont (L) : ELEM (L) → VALUE yields curr value of list elems
- initialize(L) for L ∈ LINKED-LIST (as usual, L is suppressed) as empty linked list with values in VALUE, defined as follows:
  - ELEM := { Head, Tail }   next (Head) := Tail   previous (Tail) := Head
  - previous (Head) := next (Tail) := null (ELEM)   Head/Tail start/end the list
  - cont (Head) := cont (Tail) := null (VALUE)   Head/Tail have no content



Double Linked Lists : Definition of Operations (1)

- CreateList (VALUE) ≡
  let L = new (LINKED-LIST (VALUE)) in initialize (L)
- Append (L, Val) ≡
  let e = new (ELEM (L)) in
    Link previous (Tail) & e
    Link e &Tail
    cont (e) := Val



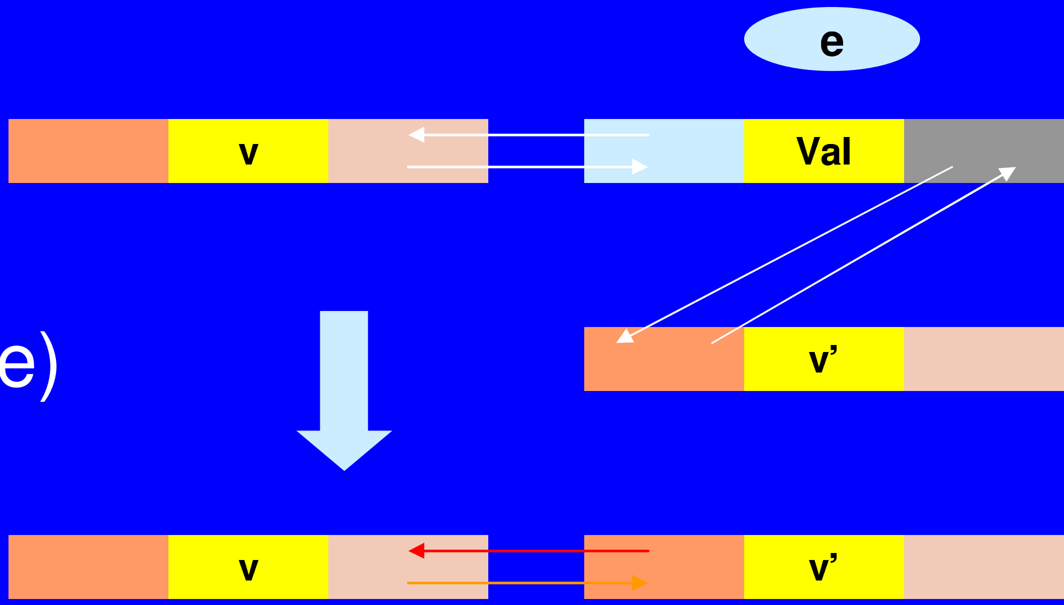
- Insert (L, Val, Elem) ≡ let e = new **(ELEM (L))** in
    cont (e) := Val
    Link Elem & e
    Link e & next (Elem)

with Link a&b ≡ next (a) := b
previous (b) := a

Double Linked Lists : Operations & Derived Fcts (2)

Delete (L, e) $\equiv$

Link previous (e) & next (e)



length (L) $\equiv \iota\, m$ ($next^{m+1}$ (Head) = Tail) well defined by initialization

occurs (L, e) $\equiv \exists\, i \leq$ length (L) : $next^{i}$ (Head) = e (e $\in$ ELEM(L))

position (L, Elem) $\equiv \iota m$ ($next^{m}$ (Head) = Elem) if occurs (L, Elem)

AccessByIndex (L, i) $\equiv$ $next^{i}$ (Head) if i $\leq$ length (L)

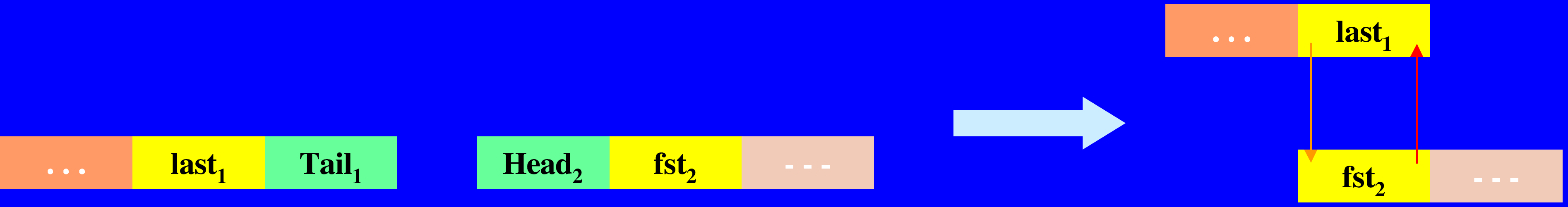
AccessByValue (L, Val) $\equiv$ $next^{m}$ (Head) fst occ of Val

where m= min { i | cont ($next^{i}$ (Head)) = Val } is defined

Double Linked Lists : Definition of Operations (3)

Update (L, Elem, Val) ≡ If occurs (L, Elem)
then cont (Elem) := Val
else error msg “Elem does not occur in L“

Cat (L_1, L_2) ≡ let L = new (LINKED-LIST) in
Head (L) := Head (L_1)
Tail (L) := Tail (L_2)
Link (L) previous (L_1) (Tail (L_1)) & next (L_2) (Head (L_2))
forall e∈ELEM (L_1) - {previous (L_1) (Tail (L_1)),Tail (L_1) }
Link (L) e & next (L_1) (e)
forall e∈ELEM (L_2) - { Head (L_2) ,Tail (L_2) }
Link (L) e & next (L_2) (e)



Double Linked Lists : Definition of Operations (4)

Split $(L, e, L_1, L_2) \equiv$ let e_1 = new-tail, e_2 = new-head

where e' = new-tail/head ≡

cont (e') **:= null (VALUE)**

next/previous (e') **:= null (ELEM)**

Head (L_1) := Head (L)

Tail (L_1) := e_1

Link (L_1) e & e_1

forall $E \in$ ELEM (L) if position $(L, E) <$ position (L, e)

then Link (L_1) E & next (L) (E)

Head (L_2) := e_2

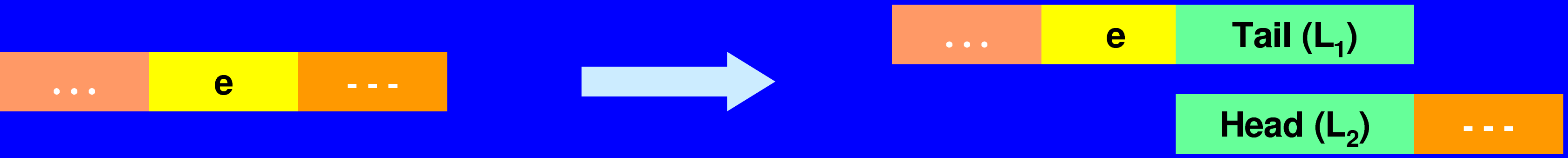
Link (L_2) e_2 & next (L) (e)

Tail (L_2) := Tail (L)

forall $E \in$ ELEM (L)-{Tail(L)}

if position $(L, e) <$ position (L, E)

then Link (L_2) E & next (L) (E)



. . . **e** - - -

. . . **e** **Tail (L_1)**

Head (L_2) - - -

Double Linked Lists : Proving the Properties (1)

- If the next-link of a list element Elem points to Elem', then the previous-link of Elem' points to Elem.
  - Initially true by defn of initialize (L), preserved by each opn due to the defn of Link (L) and the fact that next/previous are modified only using this macro.
- L is empty iff

  the next-link of its head points to its tail.
- A newly created linked list is empty and

  its length is 0.
- After applying Append (L, Val), the list is not empty
- By Append/Delete the list length in/de-creases

  by 1.

Double Linked Lists : Proving the Properties (2)

- For L≠[ ]: Append (Delete (L,E),E) = Delete (Append (L,E),E)
  - Follow from the defn of initialize (L), length (L), Append, Delete & the fact that Append/Insert yield a non null cont.
- The set ELEM (L) of elements occurring in a list is the set of all E which can be reached, starting from the list head, by applying next-links until the list tail is encountered.
  - Follows from the defn of ELEM(L).