Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 14

**Exercise 43:** [standard combinators]

Prove the following equations to be valid for the standard combinators $I \equiv \lambda x.x$, $K \equiv \lambda xy.x$, $B \equiv \lambda xyz.x(yz)$, $K_* \equiv \lambda xy.y$, $S \equiv \lambda xyz.xz(yz)$:

1. $IM = M$, 2. $KMN = M$, 3. $K_*MN = N$, 4. $SMNL = ML(NL)$, 5. $BLMN = L(M(N))$

**Exercise 44:** [Number presentations]

Let the following number presentations in the $\lambda$-calculus be given:

1. $c_0 \equiv \lambda fx.x$, $c_{n+1} \equiv \lambda fx.f^{n+1}(x)$
2. $d_0 \equiv I$, $d_{n+1} \equiv [\text{false}, d_n]$
3. $z_0 \equiv KI$, $z_{n+1} \equiv SBz_n$,

where $F^0(M) \equiv M$, $F^{n+1}(M) \equiv F(F^n(M))$, true $\equiv K$, false $\equiv K_*$, $[M,N] \equiv \lambda z.zMN$.

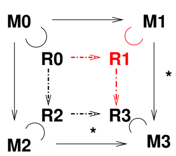
Prove:

1. There are terms $T, T^{-1}$ with $Tc_n \equiv d_n$ and $T^{-1}d_n \equiv c_n$ for all $n$.
2. There are terms $R, R^{-1}$ with $Rd_n \equiv z_n$ and $R^{-1}z_n \equiv d_n$ for all $n$.

**Exercise 45:** [properties of redexes]

1. Make yourself familiar with the notation used in chapter 11 of the lecture. Use the following paper: Bergstra, Klop :: Conditional Rewrite Rules: Confluence and Termination. JCSS 32 (1986)
2. Prove the following Lemma (Lemma 11.5 on slide 361).

   Let $D$ be an elementary reduction's diagram for orthogonal systems, $R_i \subseteq M_i$ $(i = 0, 2, 3)$ redexes with $R_0 -.-. \rightarrow R_2 -.-. \rightarrow R_3$ i.e $R_2$ is Rest of $R_0$ and $R_3$ is Rest of $R_2$. Then there is a unique redex $R_1 \subseteq M_1$ with $R_0 -.-. \rightarrow R_1 -.-. \rightarrow R_3$, i.e.

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