Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 9

Exercise 31: [Example confluence and critical pairs]

Consider the rule system $R : h(x, f(x)) \to c, h(x, x) \to b, k(x) \to x, g(a) \to f(g(k(a)))$.

1. Prove: There are no critical pairs of R.
2. Prove: R is not confluent.
3. Why is there no contradiction?

Exercise 32: [Local coherence and critical pairs]

Prove: Let $\mathrm{CP}(R, G)$ be defined as the set of critical pairs regarding R and the set of equations G oriented in both ways. If R is left-linear, then the following statements are equivalent.

1. $\to_R$ is locally coherent modulo $\sim$.
2. For every critical pair $(t_1, t_2) \in \mathrm{CP}(R, G)$ holds $t_1 \downarrow_\sim t_2$.

Exercise 33: [Termination]

Prove the following theorem:

Let A be a set, $>$ a total well-founded ordering on A and I a function mapping every k-ary function symbol f to a mapping $I(f) : A^k \to A$, strictly monotonously increasing in every argument (i.e. for all $a_1, \ldots, a_k \in A, i \in \{1, \ldots, k\}$, and $a_i > a$ holds: $I(f)(a_1, \ldots, a_i, \ldots, a_k) > I(f)(a_1, \ldots, a_{i-1}, a, a_{i+1}, \ldots, a_k)$.

Let $I(\beta) : \mathrm{Term}(F, V) \to A$ be defined as:

$$I(\beta)(t) = \beta(t), \text{ if } t \in V$$
$$I(\beta)(f(t_1, \ldots, t_n)) = I(f)(I(\beta)(t_1), \ldots, I(\beta)(t_n)).$$

Let G be a term-rewriting system and let $I(\beta)(l) > I(\beta)(r)$ for every rule $l \to r \in G$ and for every variable assigment $\beta : V \to A$. Then G is terminating.

Exercise 34: [Example for termination]

Consider the rule system $R : f(x) \to h(s(x)), h(0) \to h(s(0))$ with $x \in V$. Prove:

1. The theorem of exercise 33 is not applicable to R for $A = \mathbb{N}$.
2. R is confluent.
3. R is terminating.

Delivery: until 2009/06/01 Fr:G07 Mo:G05,
by EMail to madlener@informatik.uni-kl.de