1 Das deduktive System F_0

1.1 Axiome und Regeln

$$Ax1: \quad A \rightarrow (B \rightarrow A)$$

$$Ax2: \quad A \rightarrow (B \rightarrow C) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$$

$$Ax3: \quad (\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)$$

$$MP: \quad \frac{A, A \rightarrow B}{B}$$

1.2 Deduktionstheorem

$$\Sigma, A \vdash_{F_0} B \text{ gdw. } \Sigma \vdash_{F_0} (A \rightarrow B)$$

1.3 Theoreme

$$1: \quad \vdash_{F_0} (A \rightarrow A)$$

$$2: \quad (A \rightarrow B), (B \rightarrow C) \vdash_{F_0} (A \rightarrow C)$$

$$3: \quad \vdash_{F_0} \neg\neg A \rightarrow A$$

$$4: \quad \vdash_{F_0} (A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$$

$$5: \quad \vdash_{F_0} (B \rightarrow ((B \rightarrow A) \rightarrow A))$$

$$6: \quad \vdash_{F_0} (\neg B \rightarrow (B \rightarrow A))$$

$$7: \quad \vdash_{F_0} (B \rightarrow \neg\neg B)$$

$$8: \quad \vdash_{F_0} (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A) \text{ und}$$

$$\vdash_{F_0} (\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)$$

$$9: \quad \vdash_{F_0} (B \rightarrow (\neg C \rightarrow \neg(B \rightarrow C)))$$

$$10: \quad \vdash_{F_0} ((B \rightarrow A) \rightarrow ((\neg B \rightarrow A) \rightarrow A))$$

$$11: \quad \vdash_{F_0} ((A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A))$$

2 Gentzen-Sequenzenkalkül

2.1 Axiome und Regeln

$$Ax1: \quad \Gamma, A \vdash_G A, \Delta$$

$$Ax2: \quad \Gamma, A, \neg A \vdash_G \Delta$$

$$Ax3: \quad \Gamma \vdash_G A, \neg A, \Delta$$

$$R_{\wedge,\vee}: \quad \frac{\Gamma, A, B \vdash_G \Delta}{\Gamma, (A \wedge B) \vdash_G \Delta} \qquad \frac{\Gamma \vdash_G A, B, \Delta}{\Gamma \vdash_G, (A \vee B), \Delta}$$

$$R_{\rightarrow}: \quad \frac{\Gamma, A \vdash_G \Delta, B}{\Gamma \vdash_G (A \rightarrow B), \Delta} \qquad \frac{\Gamma \vdash_G A, \Delta; \quad \Gamma, B \vdash_G \Delta}{\Gamma, (A \rightarrow B) \vdash_G \Delta}$$

$$R_{\neg}: \quad \frac{\Gamma, A \vdash_G \Delta}{\Gamma \vdash_G \neg A, \Delta} \qquad \frac{\Gamma \vdash_G A, \Delta}{\Gamma, \neg A \vdash_G \Delta}$$

$$R_{\wedge',\vee'}: \quad \frac{\Gamma \vdash_G A, \Delta; \quad \Gamma \vdash_G B, \Delta}{\Gamma \vdash_G (A \wedge B), \Delta} \qquad \frac{\Gamma, A \vdash_G \Delta; \quad \Gamma, B \vdash_G \Delta}{\Gamma, (A \vee B) \vdash_G \Delta}$$

3 Hilbertkalkül

3.1 Regeln

Konjunktion: $\wedge\_I: \dfrac{A, B}{A \wedge B}$ $\wedge\_E: \dfrac{A \wedge B}{A}$

Disjunktion: $\vee\_I: \dfrac{A}{A \vee B}$ $\vee\_E: \dfrac{A \vee B, \neg A}{B}$

Implikation: $\rightarrow\_E: \dfrac{A, A \rightarrow B}{B}$ (Modus-Ponens) $\rightarrow\_E: \dfrac{\neg B, A \rightarrow B}{\neg A}$ (Modus-Tollens)

Negation: $\neg\_E: \dfrac{A, \neg A}{B}$ $\neg\_E: \dfrac{\neg\neg A}{A}$

Äquivalenz: $\leftrightarrow\_E: \dfrac{A \leftrightarrow B}{A \rightarrow B}$ $\leftrightarrow\_E: \dfrac{A \leftrightarrow B}{B \rightarrow A}$

Transitivität: $\leftrightarrow\_I: \dfrac{A \leftrightarrow B, B \leftrightarrow C}{A \leftrightarrow C}$

Deduktionstheorem: $\rightarrow\_I: \dfrac{A_1, \ldots, A_n, B \vdash_H C}{A_1, \ldots, A_n \vdash_H B \rightarrow C)}$

Reductio ad absurdum: $\neg\_I: \dfrac{A_1, \ldots, A_n, B \vdash_H C, A_1, \ldots, A_n, B \vdash_H \neg C}{A_1, \ldots, A_n \vdash_H \neg B}$

Hypothetischer Syllogismus: $\dfrac{A \rightarrow B, B \rightarrow C}{A \rightarrow C}$

Konstruktives Dillema: $\dfrac{A \rightarrow B, C \rightarrow D, A \vee C}{B \vee D}$